Blocking Pebbles

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University of New England
Biddeford, Maine

CGTC2, 2017

- Graph pebbling

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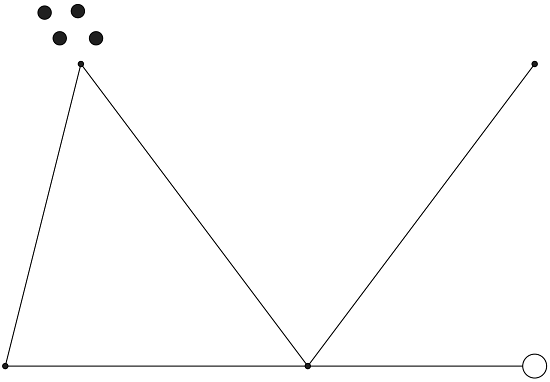

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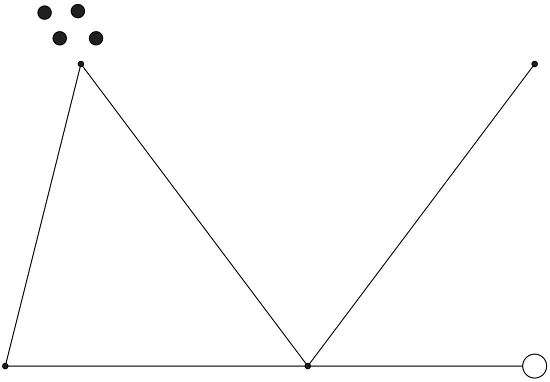

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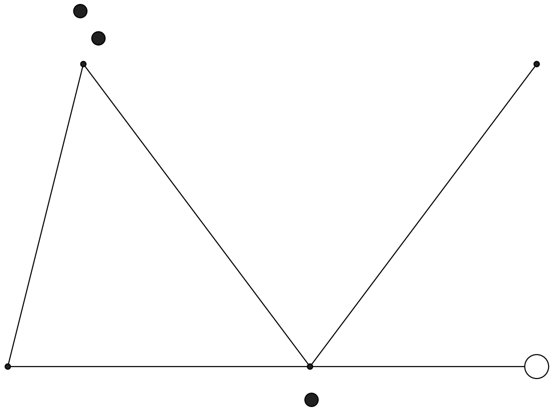

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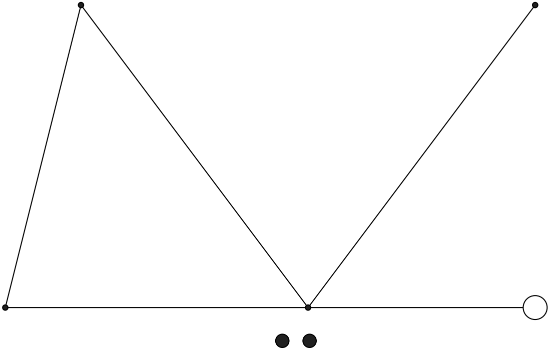

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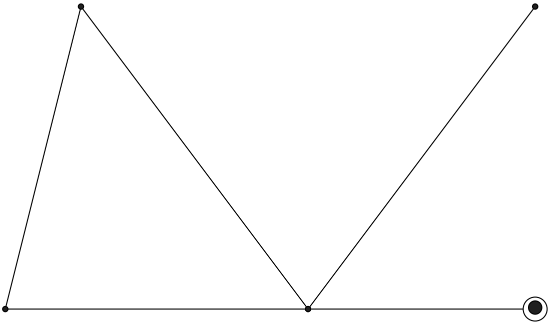

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Applications in number theory, resource allocation

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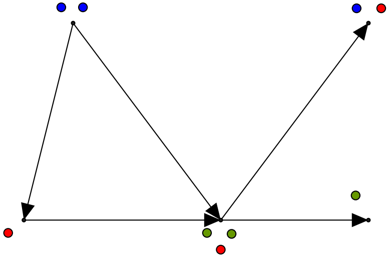

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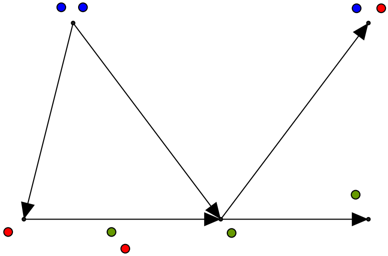

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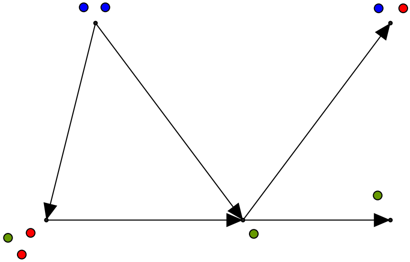

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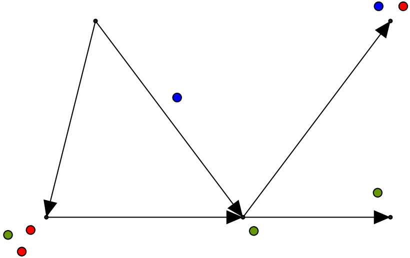

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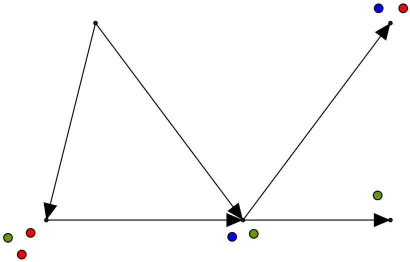

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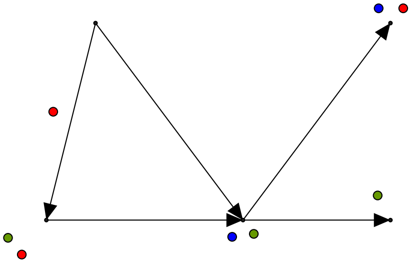

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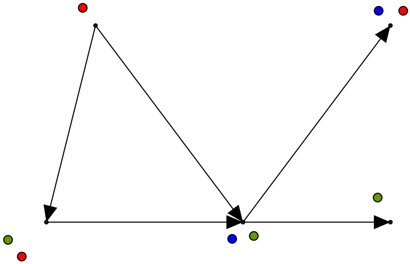

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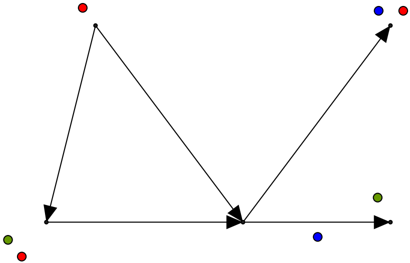

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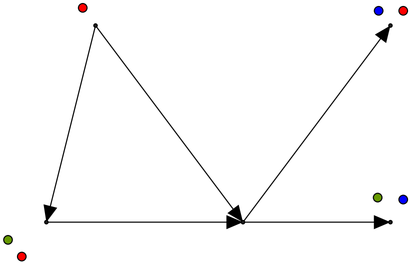

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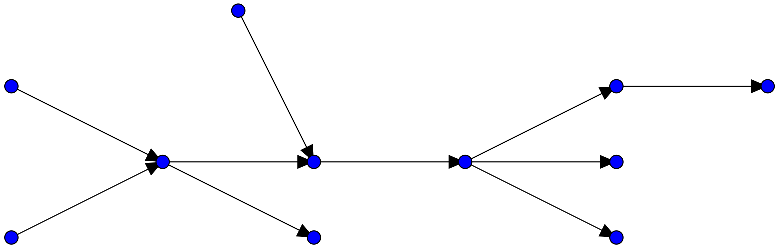

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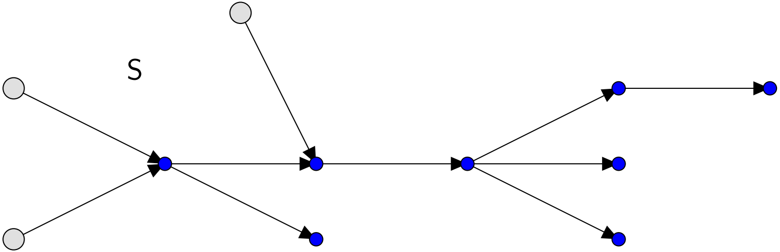

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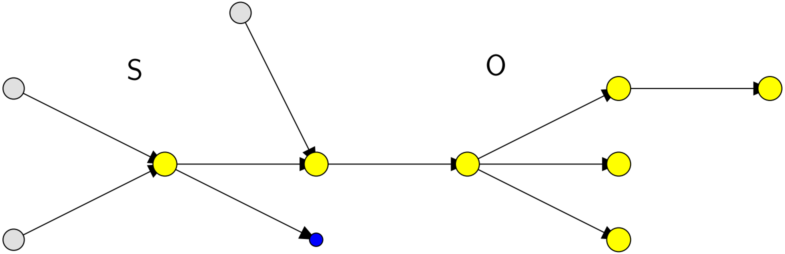

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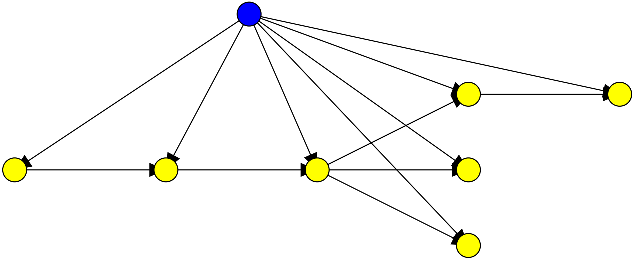

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Consider a counter-example with lowest sum

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Reduce the path to a star. □

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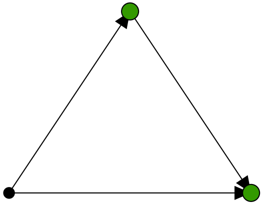

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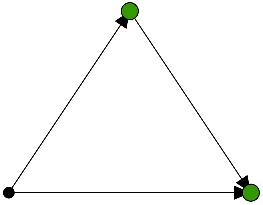

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Value of G?

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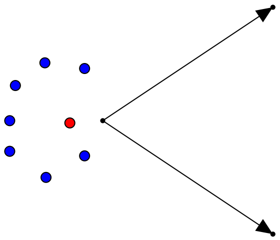

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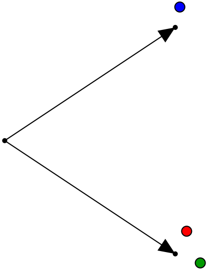

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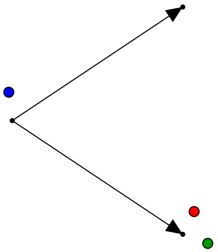

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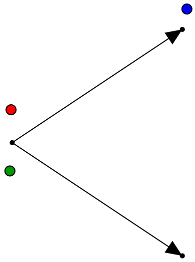

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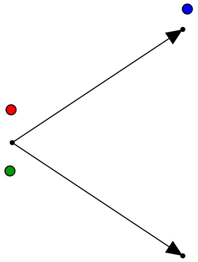
- Switches
- Lots of stars, paths



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