Eternal Picaria

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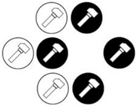
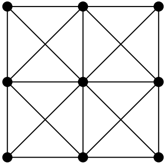


Joint work with Urban Larsson

- Picaria is a traditional board game, played by the Zuni tribe of the American Southwest and other parts of the world, such as in a rural Southwest region in Sweden
- Also known as “luffarschack”
- The co-author played this game as a child with his grandparents in the village Rångedala close to the Swedish city Borås

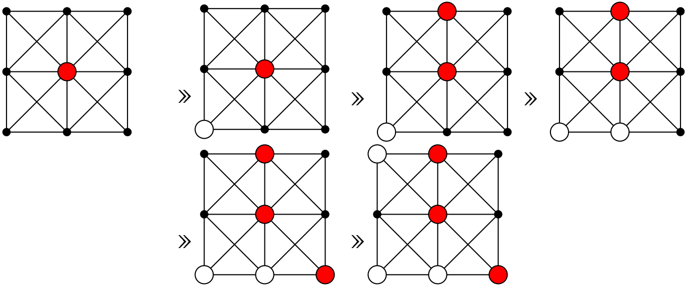


Game Rules

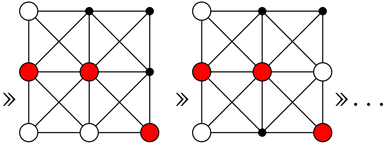
- There are two players who alternate turns
- Each player has their own type of pieces, say, X and O
- Each player has three pieces
- The goal is to be the first player to place 3 game pieces of a kind in a row, vertically, horizontally or diagonally
- The game has two phases:
- **Phase 1**: The players alternate turn to place their stones on the board
- When each piece has been played in the first phase (and assuming a non-loss so far), then phase two starts
- **Phase 2**: Player X begins by sliding one of the three Xs to an empty adjacent node
- Then O slides a stone, and so on.

Example

First phase: Placing stones



Second Phase: Sliding



Let's play!

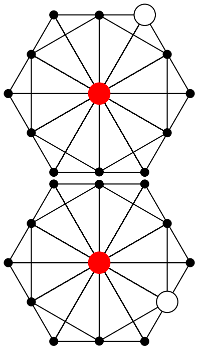
Picaria is a draw.

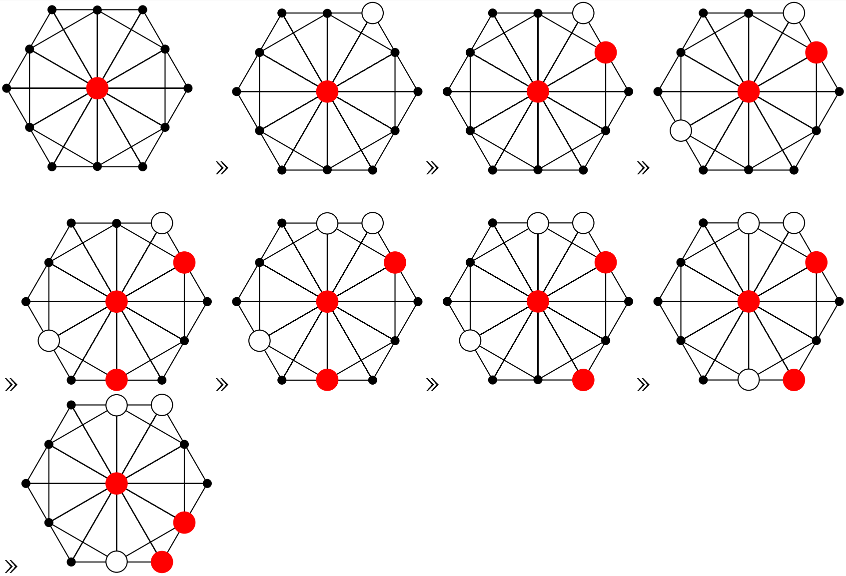
However, in a bigger board Picaria is a first player win.

A winning strategy for the first player:

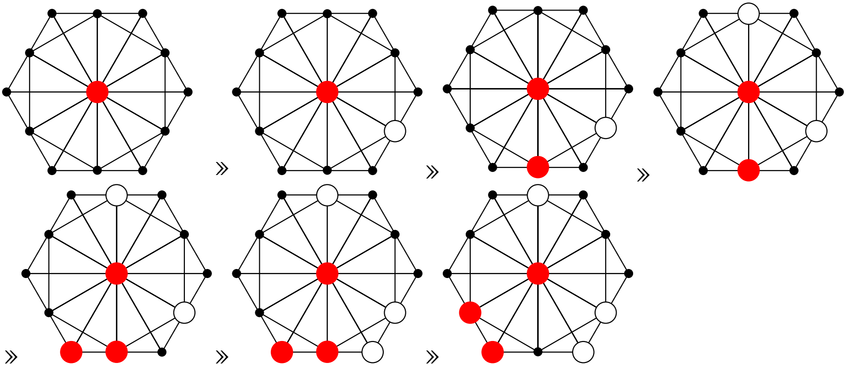
Start playing in the center!

Then, there are two options for the second player



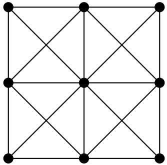


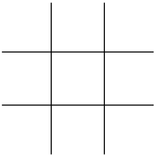
. . . and white has no chance. Red wins!



. . . white has no more chance. Red wins!

- Picaria was described in the literature for the first time in 1907 by the ethnographer Stewart Culin (Games of North American Indians. Washington DC: US gov Printing Office. 846 pp. (1907).)
- In our paper we describe the game in an equivalent manner using instead a Tic-tac-toe board





- To use the convention of Tic-tac-toe, we use X and O to indicate the pieces
- In our study, we assume that player X starts

	X	O

$\xrightarrow{x}$

	X	O
X		

$\xrightarrow{o}$

		O
	X	O
X		

$\xrightarrow{x}$

		O
	X	O
X		X

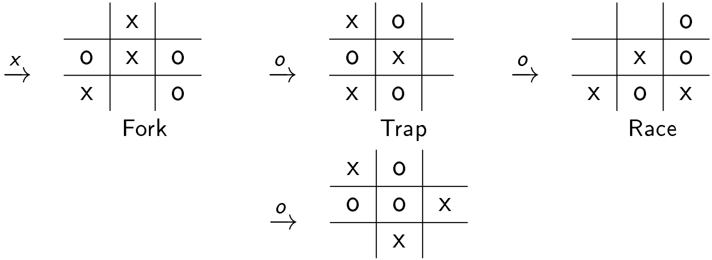
$\xrightarrow{o}$

		O
	X	O
X	O	X

- Before all stones are on the board, the number of positions coincide with those of Tic-tac-toe
- Applying Burnside Lemma we can count the number of orbits that those positions belong
- There are 450 positions modulo symmetries
- Even though a computer could be used to solve Picaria, this would not provide a full understanding on how to play a succesful strategy

- The ‘blockade’ games of Pong Hau K’i, from China, and Mu Torere, played by the Māori people from the east coast of New Zealand’s North Island, are also related (Philip D. Straffin, Jr. Position Graphs for Pong Hau K’i and Mu Torere. Mathematics Magazine Vol. 68, No. 5 (Dec., 1995), pp. 382-386.)
- In these games, if a player cannot move, he loses
- With quite few positions, only 16 and 46 respectively, these games were solved in the literature by depicting the position graphs

- We give the explicit strategies of optimal play for Picaria
- It turns out that both players are able to draw from the initial position
- Efficient strategies to win by for example using Fork-, Trap-, Race-, or Zugzwang-positions, will not occur in optimal play



Fork Trap Race

Zugzwang

- First we prove that O cannot win
- To prove that X cannot win is more delicate
- One of the key ideas:
- We analyse a special configuration which is quite recurring in the game, called a Loop position
- We show that O prevents X from winning the game by means of a periodic sequence of moves

$\xrightarrow{o}$

		o
o	x	x
x		o

Loop

- Another key idea:
- When Picaria is played by human (non-optimal) players, a player holding the center often appears to enjoy a certain advantage
- Consider that player X holds the center
- This restricts the possibilities for X in that only the two outer stones can be moved
- In particular if X starts from the Loop position, then player X cannot win, which constitutes our next lemma

Lemma

If player X is to move and it refuses to leave the center starting from a Loop position, then player O can force a return to this position.

Proof

		O
O	X	X
X		O

Loop

There are only three possible moves from a Loop-position for X, since it holds the center.

		O
O	X	
X	X	O

(A)

	X	O
O	X	
X		O

(B)

		O
O	X	X
	X	O

(C)

For game (A), player X gets trapped by

		o
o	x	
x	x	o

(A)

$\xrightarrow{o}$

o	x	o
x	x	o

Therefore, the game (A) does not belong to X's strategy.

		O
O	X	X
	X	O

(C)

$\xrightarrow{o}$

	O	
O	X	X
	X	O

Now by symmetry X moves to

$\xrightarrow{x}$

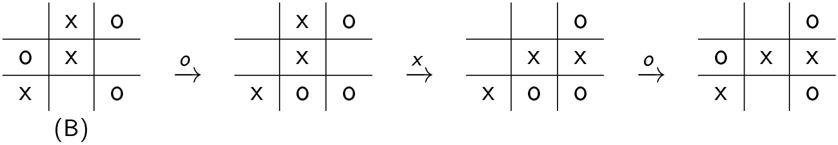
	O	X
O	X	
	X	O

$\xrightarrow{o}$

	O	X
	X	
O	X	O

which returns to the initial Loop-position.

Finally, position (B).
Player O can force the following sequence



which is the initial (Loop) position.

□

- The next result deals with the most delicate position.
- Basically, all interesting positions arise here.

Lemma

For game

		o
	x	

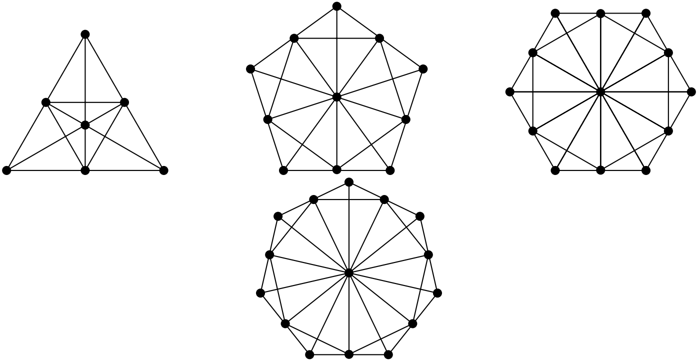
X cannot win.

Thus, we obtain

Theorem

Player X cannot win.

Consider the following natural generalizations of Picaria. We use the same set of rules and only change the number of sides of the board as depicted.



- Following this pattern, there are infinitely many different board games in this family
- One can show that all these games are first player win in a few moves, except for Picaria

Open questions

- What happens if we increase the number of stones for each player, say that game parameters, $k \geq 3$ stones each and $s \geq 3$ sides, are given (otherwise the same rules).
- Is there any combination (k, s), other than $(3, 4)$, for which the game is a draw (provided that the total number of stones is less than the number of nodes)? Is it true that the second player never wins?
- If we give the second player a one stone advantage (handicap), for which combination (k, s) can he draw/win the $((k, k+1), s)$ game (that is the second player places his last placement stone after the first slide-along-edge move by the first player)?
- In general, how many stones advantage $l > 0$ does he require to draw/win a generalized Picaria?

Thank you!