NEW RESULTS IN CIRCULAR NIM

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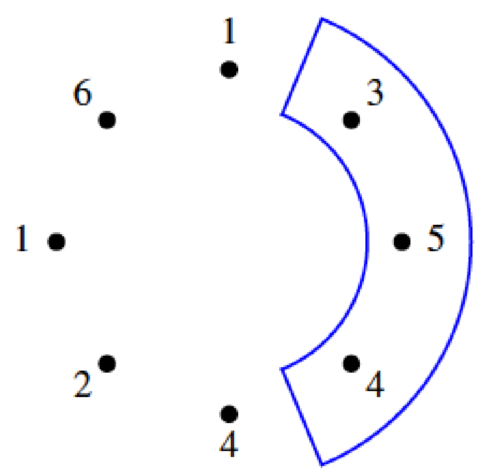
Definition of the Game CN(n,k)

- n stacks of tokens arranged in a circle
- Select k **consecutive** stacks and remove at least one token from at least one of the stacks
- Last player to move wins



$k = 1$ corresponds to regular Nim

General Results

Game	P-Positions
CN(n,1) = Nim	$\{(p_1, p_2, \ldots, p_n) \mid p_1 \oplus p_2 \oplus \cdots \oplus p_n = 0\}$
CN(n,n)	$\{(0, 0, \ldots, 0)\}$
CN(n,n-1)	$\{(a, a, \ldots, a) \mid a \geq 0\}$

- These cover all cases for n = 1, 2, 3 and all but CN(4,2).
- The set of P-positions for CN(4,2) is given by $\{(a,b,a,b) \mid a, b \geq 0\}$

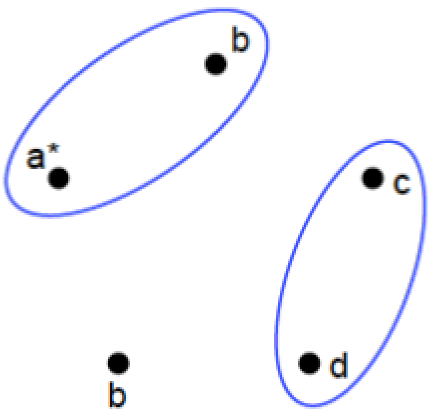
Results for n = 5

Theorem [D; Ehrenborg and Steingrimsson]

The P-positions of CN(5,2) are given by

$\{(a^*, b, c, d, b) \mid a^* + b = c + d,\ a^* = \max(\mathbf{p})\}$

Note that $b = \min(\mathbf{p})$

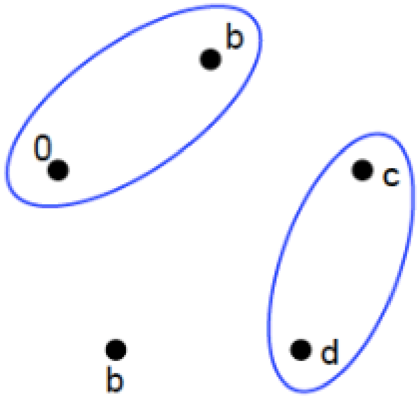


Theorem [Ehrenborg and Steingrimsson]

The P-positions of CN(5,3) are given by

$\{(0, b, c, d, b) \mid b = c + d\}$

Note that $b = \max(\mathbf{p})$

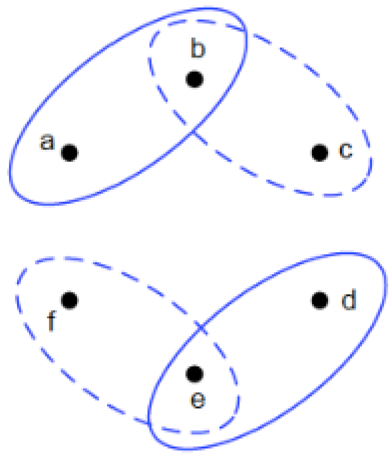


Results for $n = 6$

Theorem [DH]

The P-positions of CN(6,3) are given by

$\{(a,b, c, d, e,f) \mid a + b = d + e,\ b + c = e + f\}$



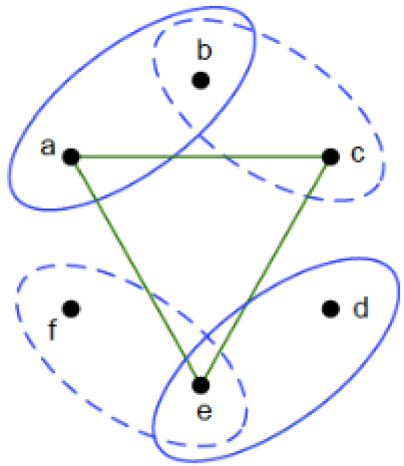
Note that also $c + d = f + a$

Theorem [DH]

The P-positions of CN(6,4) are given by

$\{(a,b, c, d, e,f) \mid a + b = d + e,\ b + c = e + f,$

$a \oplus b \oplus c = 0,\ a = \min(\mathbf{p})\}$



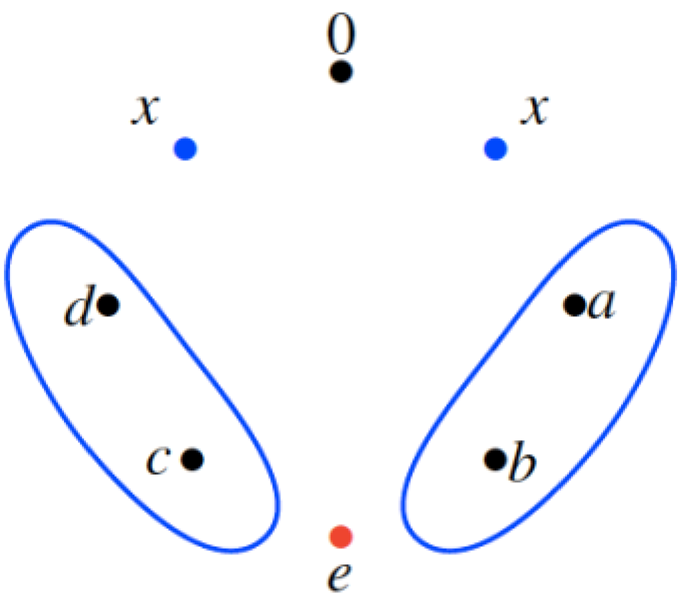
Note that also $c + d = f + a$

Results for $n = 7$ (new)

Results for *n* = 8

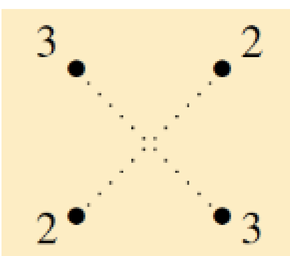
Theorem [DH]

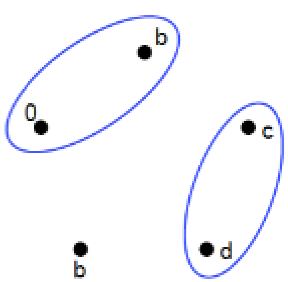
The P-positions of CN(8,6) are given by {(0, x, a, b, e, c, d, x)| a + b = c + d = x, e = min { x, a + d }}

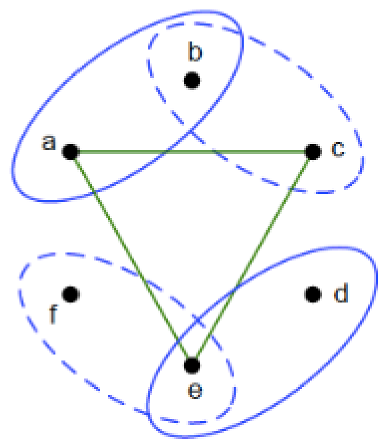


Open Questions

- Missing cases are CN(6,2), CN(7,2), CN(7,5)
- For n = 8, only CN(8,6) is known
- It seems that CN(n,2) are difficult to find
- Also, CN(n,n-2) do not show common structure

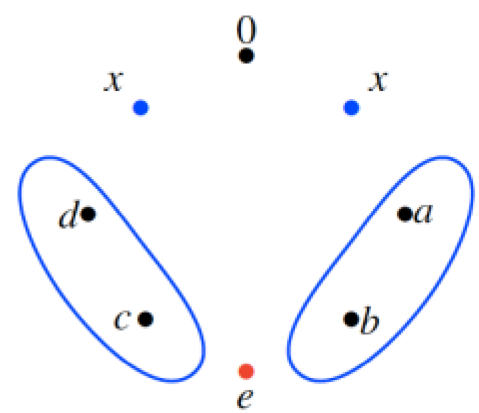






a = min(**p**)

?



Obrigado!